

MONDAY, MAY 15, 1961  
SESSION 2: PARAMETRIC DEVICES

2:00 PM - 4:45 PM  
CHAIRMAN: W. W. MUMFORD  
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## 2.1 TRANSMISSION PHASE RELATIONS OF FOUR-FREQUENCY PARAMETRIC DEVICES

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The transmission phase properties of parametric amplifiers are tantamount to gain, bandwidth, excess noise temperature, and pump power in angular detection systems such as monopulse radars and interferometers. These angle detection systems employ antenna and hybrid junction labyrinths which derive even and odd spatial components of the antenna diffraction pattern. The labyrinth ports usually supply even, elevation odd, azimuth odd, and quadrupolar components. The source signal or target echo, differential amplitude and differential phase, between labyrinth ports contain spatial information of position, extent, and shape of the target. Consequently, to employ the potential of parametric devices in monopulse detection systems requires an understanding of the transmission phase properties of parametric devices. It is the purpose of this paper to delineate these properties.

The analysis follows the matrix representation of a nonlinear capacitive reactance, four-frequency device wherein due note is taken of the phases. The phase properties are analyzed for the four-frequency devices because they encompass both negative resistance and frequency conversion amplification mechanisms. The four-frequencies considered are:

$$\omega_1 < \omega_2 = \omega_p - \omega_1 < \omega_p < \omega_3 = \omega_p + \omega_1$$

The expressions for transmission phase include the effects of non-zero port susceptances and nonlinear reactance element losses. At midband without losses, the equations reduce to easily-remembered equations which are significant to application of parametric devices in phase-sensitive systems and distinguish the parametric process from conventional nonlinear resistive frequency mixing. At midband without losses the transmission phase relations are:

Input to Output	Phase Relation
$\omega_1 \rightarrow \omega_2$	$\phi_2 = \phi_p - \phi_1 - \pi/2$
$\omega_2 \rightarrow \omega_1$	$\phi_1 = \phi_p - \phi_2 - \pi/2$
$\omega_1 \rightarrow \omega_3$	$\phi_3 = \phi_p + \phi_1 - \pi/2$
$\omega_3 \rightarrow \omega_1$	$\phi_1 = \phi_3 - \phi_p - \pi/2$
$\omega_2 \rightarrow \omega_3$	$\phi_3 = 2\phi_p - \phi_2 - \pi$
$\omega_3 \rightarrow \omega_2$	$\phi_2 = 2\phi_p - \phi_3$

The sign of  $\pi/2$  applies for a nonlinear capacitive reactance and will change for a nonlinear inductance. These phase relations are to be distinguished from the relations for the propagation constants for traveling wave structures. The simple transmission phase relations above will be generalized to include the effects of nonzero port susceptances and nonlinear reactance element losses by including an arctangent function. The dependence upon diode cutoff frequency, circuit Q's, mode spectrum frequency, and deviation from midband is shown.

These transmission phase relations show that the non-inverting up-conversion process of parametric amplification differs from other parametric processes such as non-inverting down-conversion and the inverting negative conductance amplifier. By way of illustration, the transmission phase relations are applied to show nonreciprocal or unilateral properties of certain parametric devices.

When parametric devices are used in monopulse systems, the transmission phase relations have proved useful in system analysis in the same manner as the transmission phase rules distinguish differences between the various microwave hybrid junctions used in the design of monopulse labyrinths. Parametric devices that are used to amplify and process spatial position and extent information in monopulse systems must preserve the coherence between the even and odd channels. The basic mechanisms to preserve the coherence between the even and odd channels. The basic mechanisms to preserve coherence together with experiments are indicated.

